About the Novosibirsk function

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The Novosibirsk function is often used to fit energy spectra from electromagnetic calorimeters. In particular, when the physical spectrum contains only one energy line, the detector signal distribution shows a broad, asymmetric peak with a tail towards the low signal values. The width of the peak is determined by the finite energy resolution of the calorimeter, whereas the tail is due to the energy losses caused mainly by shower leakages. In the limit of no energy losses the expected, signal distribution is a gaussian.
For a real detector, the energy losses are finite and the gaussian distribution can not be used to fit the data, in order to extract the peak position and width. Therefore, another function is needed and a typical candidate is the Novosibirsk function. It is defined as

$$f(x) = A \exp\left\{ -\frac{1}{2s^2} \log^2(1-Y) - \frac{1}{2}\, s^2 \right\} \,, \tag{1}$$

where

$$Y = \frac{x-\mu}{\sigma}\, \eta \,, \tag{2}$$

$$s = \frac{2}{\xi}\, \text{asinh}\left[\frac{\eta\,\xi}{2}\right] \,. \tag{3}$$

The expression (1) depends on the four parameters A, μ, σ and η, which are fit to the data[1]. The constant ξ in eq. (3) is the ratio between the full width at half maximum (FWHM) and the standard deviation for a gaussian curve:

$$\xi = 2\,\sqrt{2\,\log 2} \simeq 2.35482 \tag{4}$$

Meaning of the parameters

The parameter μ gives the position of the peak, as can be verified taking the first derivative of $f(x)$

$$f'(x) = f(x) \cdot \left[\frac{\log(1-Y)}{s^2(1-Y)}\, \frac{\eta}{\sigma} \right] \,, \tag{5}$$

which vanishes for $Y = 0$, i.e. $x = \mu$.
Parameter A is related to the peak height H by the expression

$$H = f(x=\mu) = A\, e^{-s^2/2} \,. \tag{6}$$

The width of the peak is determined by the parameter σ, which is used to define the energy resolution. For a gaussian spectrum, the relative energy resolution is conventionally defined as the ratio between the standard deviation and the mean of the distribution. These would not be appropriate parameters in the case of the Novosibirsk function, because of the tail of $f(x)$. It is preferable to use the position

[1]Sometimes a fifth parameter C is also used in the fit as additive constant to eq. (1) for parametrising an energy-independent background.

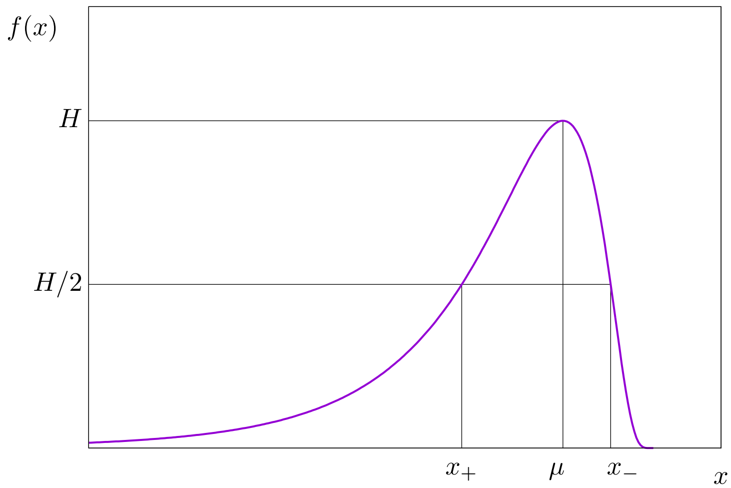

Figure 1: Illustration of the Novosibirsk function for $\eta > 0$.

of the peak maximum and the FWHM instead. To be in line with the gaussian case, the relative energy resolution is defined as σ/μ, since from the definition of $f(x)$, the relation

$$\sigma = \frac{\text{FWHM}}{\xi} \tag{7}$$

holds. In order to prove eq. (7), the abscissae of the half maximum need to be determined solving the equation:

$$\frac{H}{2} = \frac{A}{2}\, e^{-s^2/2} = f(x)\ . \tag{8}$$

Dividing by $A\, e^{-s^2/2}$ and taking the logarithm, one has

$$\log^2(1 - Y) = 2s^2 \log 2\ . \tag{9}$$

Taking the squared root, two equations emerge, which can be expressed in terms of the two solutions $Y_\pm$:

$$\log(1 - Y_\pm) = \pm\sqrt{2\log 2}\ s = \pm\frac{\xi}{2}\ s\ . \tag{10}$$

Exponentiating and solving for the sought abscissae $x_\pm$, one gets

$$x_\pm = \frac{\sigma}{\eta}\left(1 - e^{\pm\,\xi s/2}\right) + \mu\ . \tag{11}$$

The FWHM is defined as $|x_+ - x_-|$. Recalling that

$$\sinh x = \frac{e^{\,x} - e^{-x}}{2} \tag{12}$$

and noticing that

$$\frac{\xi s}{2} = \text{asinh}\left[\frac{\eta\,\xi}{2}\right]\ , \tag{13}$$

the FWHM reduces to ($\sigma > 0$ is assumed)

$$\text{FWHM} = \left|\frac{\sigma}{\eta}\ 2\ \sinh\frac{\xi s}{2}\right| = \xi\,\sigma\ . \tag{14}$$

A graphical illustration of the parameters of the Novosibirsk function discussed so far is shown in fig. 1.

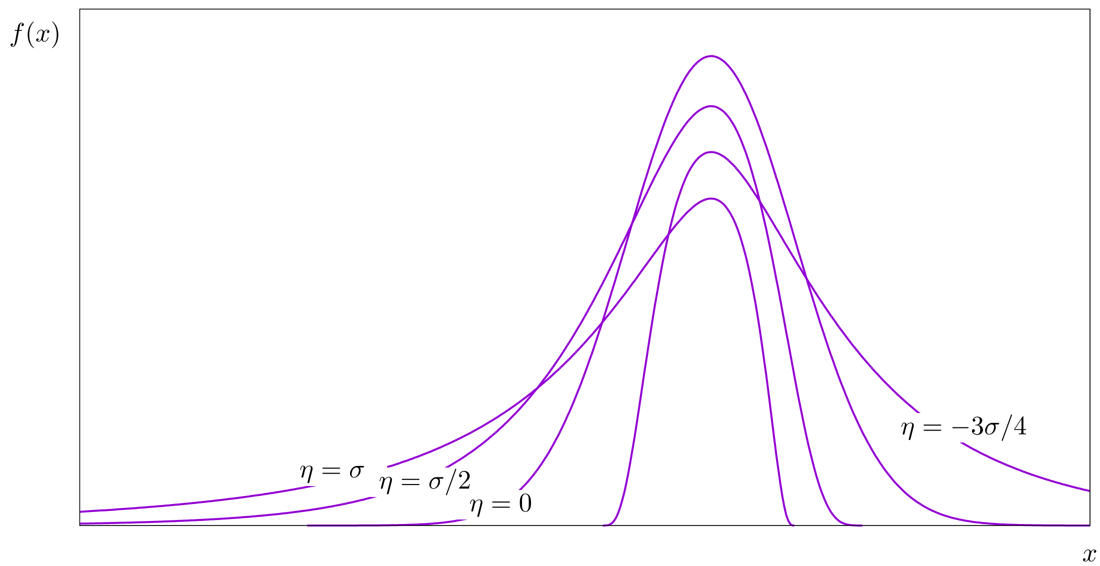

Figure 2: Effect of the parameter η on the tail of the Novosibirsk function.

The last parameter η controls the tail of the distribution. In fig. 2, the function $f(x)$ is plotted for various values of this parameter. For $\eta > 0$ or $\eta < 0$, the tail extends towards small or high energies, respectively.

In the limit $\eta \to 0$, the function tends to the gaussian distribution. Note that the first term at the exponent in eq. 1 is an indeterminate form "0/0" in the limit of vanishing η. Second, consider the limits

$$\begin{aligned} \log(1-Y) &\overset{\eta\to 0}{=} -\frac{x-\mu}{\sigma}\,\eta + \mathcal{O}(\eta^2)\ , \\ s &\overset{\eta\to 0}{=} \eta + \mathcal{O}(\eta^2)\ , \end{aligned}$$

which implies that

$$\left[\frac{\log(1-Y)}{s}\right]^2 \xrightarrow{\eta\to 0} \left[\frac{x-\mu}{\sigma}\right]^2 \tag{15}$$

and thus

$$f(x) \xrightarrow{\eta\to 0} A\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]\ , \tag{16}$$

i.e. the gaussian function. Note that, because of the logarithm at the exponent, $f(x)$ is not defined for all real values x (like the gaussian) but only for $Y < 1$, or

$$x \lesseqgtr \mu \pm \frac{\sigma}{|\eta|} \quad \text{for} \quad \eta \gtrless 0\ . \tag{17}$$